HADMÉRNÖK

V. Évfolyam 2. szám - 2010. június

Bugyjás József
bugyjas.jozsef@kvk.uni-obuda.hu

EFFECTS ON THE MODELING RESULTS OF INCREASING FINITE ELEMENT NUMBERS

Absztrakt/Abstract

Jelen tanulmány egy robbanás folyamatát vizsgálja. Egy olyan véges elem számítógépes modelljét mutatja be, amely a folyamat matematikai modelljére épül. A cél a modell és az elméleti eredmények összehasonlításával a lehetőségek tanulmányozása, valamint a korlátok javítása.

This study deals with the analysis of an explosion process. Model has been created by a Finite Element Model Software run on computer, according to the process of mathematical modeling. It studies the possibilities and limits for improvement of the model by comparing the theoretical and model results.

Kulcsszavak/Keywords: *véges elem, robbanás, modellezés ~ finite element numbers, explosion, modelling*

1. PROCESS OF MATHEMATICAL MODELING

In the methodes using FEM[1] a methode focusing on mathematics has been chosen for modeling the explosion process. This methode is the mathematical FEM. [1] Steps of the process are illustrated on Figure 1.

[1] FEM = Finite Element Methode

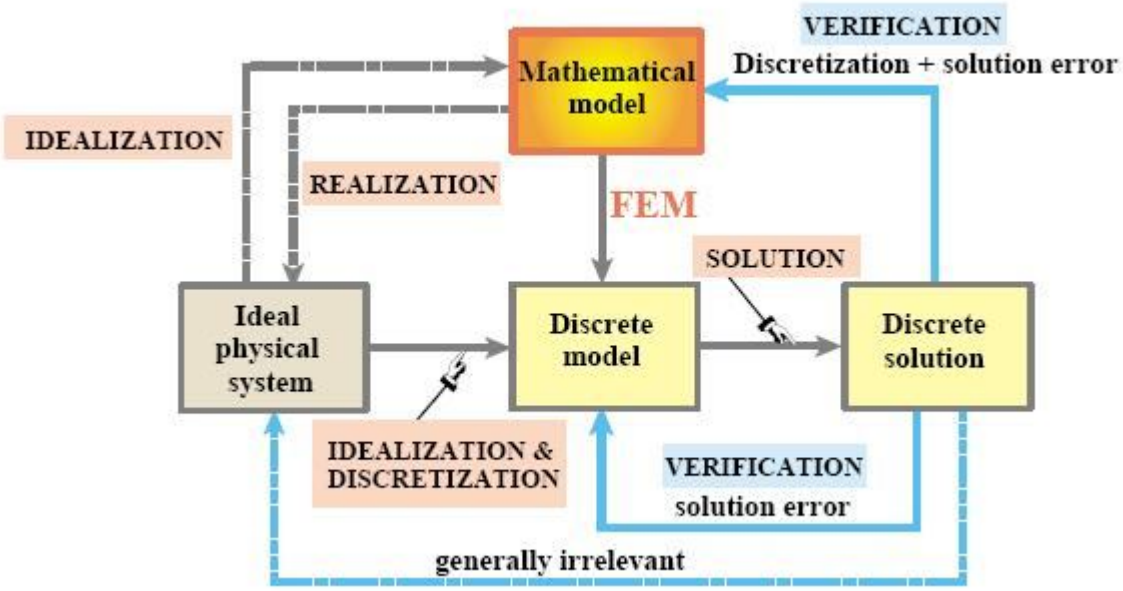


Figure 1. Mathematical FEM model [1]

Source of the proccess is mathematical model, which often is an ordinary – or partial differential equatation in space and time. A discrete finite element model is generated from a variational – or weak form of the mathematical model.

The concept of error arises when the discrete solution is substituted in the „model". This error is the amount by which the discrete solution fails to satisfy the discrete equations. This error is relatively unimportant, when using computers, and in particular direct linear equation solvers, for the solution steps.

More relevant is the discretization error, which is tha amount by which the discrete solution fails to satisfy the mathematical model.

Replacing into the ideal physical system would in principle quantify modeling errors, but this substitution in the mathematical FEM this is largely irrelevant, since the ideal physical system is merely a figment of the imagination.

2. THE MATHEMATICAL MODEL, THEORETICAL BACKGROUND

Detonation of high explosives is a mechanism when „high power" materials release their chemical energy. The chemical reaction, causing the release of energy, takes place in a narrow zone. This reaction zone propagates at high speed through the explosive that transforms the sloide explosive into hot compressed gases. This reaction zone has then a form of discontinouos wave similar to a shock wave, and has a pysical behavior which is determined solely by the properties of the unreached and completely reacted material on either side of the wave. This allows usage of the hydrodinamic approach of detonation .

Hydrodynamic Theory of Steady-state Plane Detonation [7]

The modell of the plane, steady-state reaction zone propagating at a constant speed D through the explosive is illustrated in Figure 2.

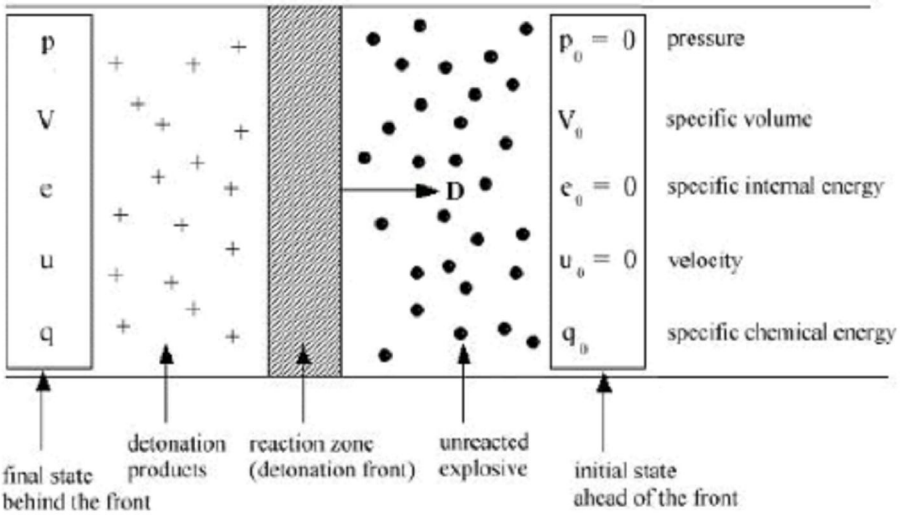


Figure 2. Plane Reaction Zone Propagating at Constant Speed

The Rankine-Hugoniot relations, which express the conservation of mass, momentum and energy in the material stream flowing through the reaction zone can be used to relate the hydrodynamic variable across the reaction zone.

Conservation of mass and momentum: $$p - p_0 = \frac{D^2}{V_0^2}(V_0 - V) \qquad (1)$$

Conservation of energy: $$e - e_0 = \frac{1}{2}(p + p_0)(V_0 - V) + q_0 \qquad (2)$$

Equation (1) describes a straight line (Rayleigt-line) determining the locus of all possible final states (p, V) , attainable by a continuous transition from the initial state (p_0, V_0) consistent with conservation of mass and momentum.

Equation (2) is merely thermodynamic, from which, with a given,equation of state p=p(V,e) relevant for the detonation products, the energy term can be eliminated , and results the Hugoniot curve of the explosive. The downward concave curve defines locus of all possible finals states (p, V) available by a discontinuous transition ffrom the initial state (p_0, V_0), consistent with conservation of energy. (Figure 3.)

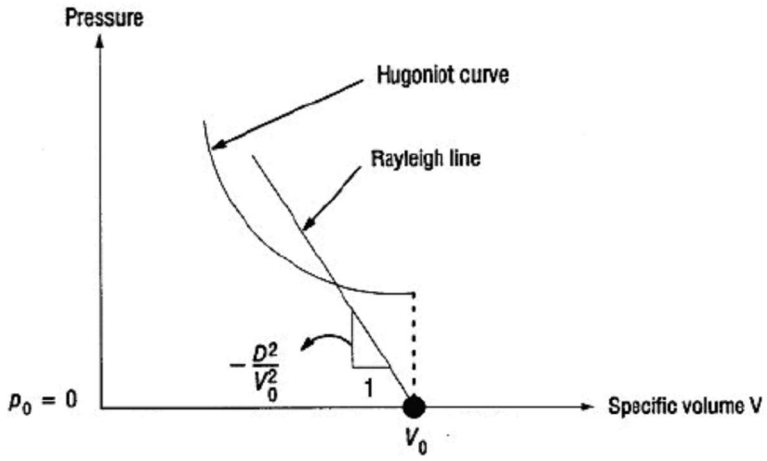


Figure 3. Hugoniot curve and Rayleigh straight

Forms of the Rayleigh straight and the Hugoniot curve are such that their interaction permits the existence of any detonation speed D above a „minimum value" and each value of D is consistent with two possible final states for the detonation products. There is one more condition required, which was supplied by Chapman and Jocquet, who added the following condition to conservation of mass, momentum and energy:

The detonation speed D is either such that the Rayleigh line is tangent to the Hugoniot curve of the explosive, or the detonation speed is the minimum velocity consistent with the Rankine-Hugoniot relations.
This process

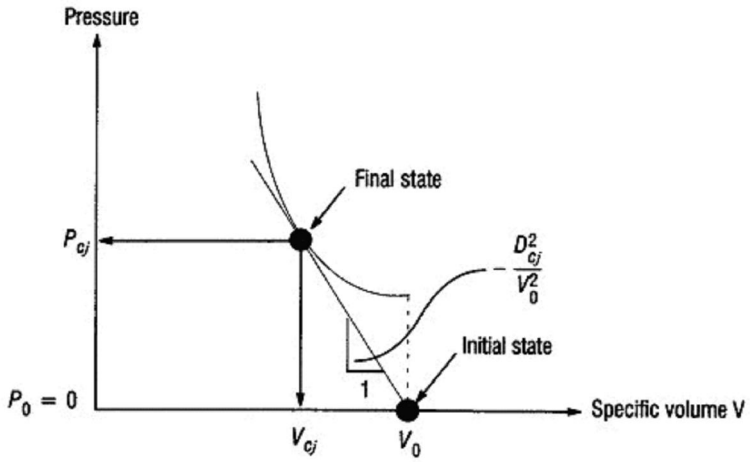


Figure 4. Hugoniot curve and Rayleigh straight in detonation process

According to the above considerations, when the ideal gas equation of state (with constant specific heat ratio γ) is to model the detonation products, the above formulas result:

$$P_{cj} = 2(\gamma - 1)q_0 p_0 \qquad (3)$$

$$V_{cj} = \frac{\gamma}{\gamma + 1} V_0 \qquad (4)$$

These relations can be applied also when using the JWL equation of state (with variable specific heat ratio) when measuring γ_{cj} at the Chapman-Joguet state (P_{cj}, V_{cj}) is behind the detonation front.

JWL Equation of State for Explosive Charges

The Jones-Wilkens-Lee (JWL) equation of state [2] is modeling the pressure, which is generated by the extension of chemical explosive material detonation. This formula is widely applied for engeenering calculations. It can be described by the bellow formula:

$$P = C_1\left(1-\frac{\omega}{R_1 v}\right)e^{-\eta v} + C_2\left(1-\frac{\omega}{R_2 v}\right)e^{-\eta v} + \frac{\omega e}{v} \qquad (5)$$

Where v is the specific volume, e is the specific energy, C_1, C_2, R_1, R_2, ω are dynamic coefficients of well-known explosive materials [3].

3. FEM PROGRAM APPLIED

MSC Dytran is a general-purpose, three-dimensional explicit finite element analysis sotware for simulating and analysing extreme short-duration events involving deformation of structural materials and the interaction of fluids and structures. For the easy modeling Dytran combine in a special way the finite element methode with mechanics of fluids.

The MSC Dytran Explicit FEA nonlinear solver technologies are used to analyse extreme, short duration, transient events within Lagrange and Euler domain. It can be used to examine modeling shocks, impacts, nonlinear lost of stability. It permits also simulation of interaction between stabile structures and fluids. MSC Dytran Models can be generated and analysed by MSC Dytran software .

The Patran is an open, user-friendly MCAE[2] application including a professional pre-and postprocessor.

4. MODELING OF CONSTANT DETONATION

For the simulation an average explosive, called Comp.B material has been chosen , which can be a reference in the future. Parameters of the detonation are included in the table 1.

	C_1 (10^{11}Pa)	C_2 (10^9 Pa)	R_1	R_2	ω	e_0 (MJ kg^{-1})	VOD (m s^{-1})	ρ_0 (kg m^{-3})
COMP.B	5.242	7.678	4.2	1.1	0.34	4.969	7980	1717

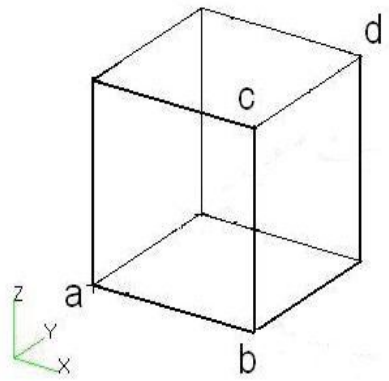
Table 1. JWl parameters of explosive used for modeling

e_0 specific chemical energy; VOD is C-J explosion speed

[2] Mechanical Computer-Aided Engineering

Target of the simulation consists in modeling the explosion of the material, verifying pressure behind the detonation front against theoretical Chapman-Jouguet value, and monitoring formation and expansion of shockwaves. [4]

The software applied uses the so-called „programmed burn" technology for the modeling of detonation of high-explosive materials. Basic condition of this technology consist in the fact, that reaction zone is constant in all directions, and it expands by the Dcj explosion speed of Chapman-Jouguet. As soon as it achieves and passes an element, the chemical energy develops proportionally on the element in the „burning time".
For the modeling of the detonation a cube of 15 mm edge was created. Nodes designed on Figure 5. are primary nodes for the tests since our further analysis will cover also the solids touching with explosive. We examined temporal procession of burning in the nodes, formation of pressure between nodes a-b, and also procession of pressure on „ab", „bc" and „cd" sections in certain moments.
The explosive charge was divided in four ways along the side edge of the cube.



Scaling:

Figure 5. Marks of explosive

1st. case 0.003 [mm] therefore 216 nodes and 125 Hex 8 elements set.
2nd. case 0.001 [mm] therefore 4096 nodes and 3375 Hex 8 elements set.
3rd. case 0.0005 [mm] therefore 29791 nodes and 27000 Hex 8 elements set.
4th. case 0.0003 [mm] therefore 132651 nodes and 12500 Hex 8 elements set.

Further reduction of the scale and therefore increase of number of elements was not possible due to limited memory capacity of the PC.

During program running following parameters and requirements wre used:
Duration of test: 5·10-6 [sec] (ENDTIME)
Data types to be displayed in 1·10-7 [sec] periods (TIMES)
Minimal and maximal intervals of analyse $1 \cdot 10^{-8}$ [sec] and $1 \cdot 10^{-7}$ [sec] (PARAM, MINSTEP; PARAM, MAXSTEP)
Hydrodynamic therories were assigned to the elements of the explosive charge (PEULER,1,1,Hydro)
Starting point of the detonation is point „a" with coordinates (0,0,0).

Geometric disposition, finite element network, material characteristics, starting conditions and requirements needed for the evaluation were given in the MSC Patran software, which generated a „.dat" extension file as a result for MSC Dytran.

Generated source file is the same in all four test cases. Discrepency was only in the „.bdf" extension file, where according to the scale other nodes and elements were used.

After running of the „.dat" extension file with MSC Dytran software, differeint type of files are generated, from which the following files can be processed further on:

.OUT MSC Dytran output file
.ARC MSC Dytran archive file, which permits MSC Patran postprocessing, that means evaluations and analysis of data.
_ERROR_SUMMARY.MSG Error messages during the FEM program running.

5. RESULTS, CONCLUSIONS

Results of the burning process

Analysis for all four cases were executed wih MSC Patran. Figures illustrating the burning process:

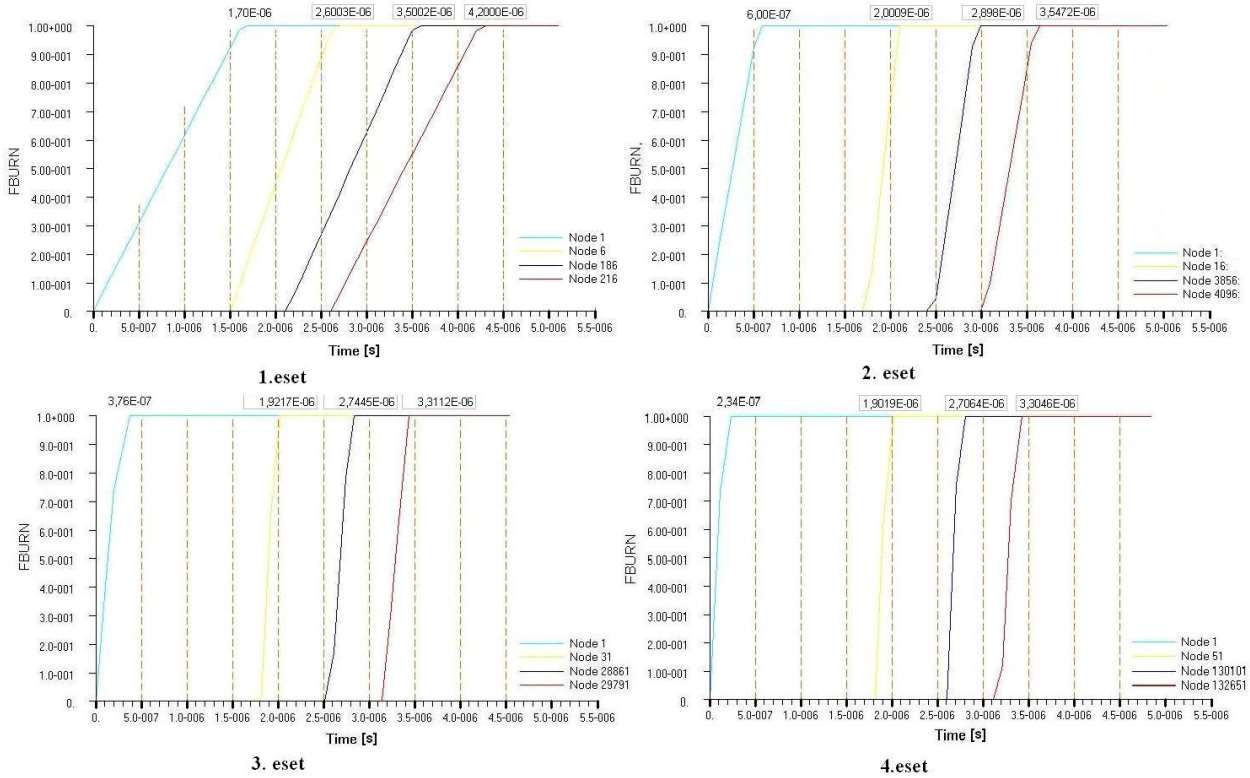


Figure 6. Burning processes of emphased nodes

Result data gave base for the examination of burning speeds,and they were compared with the theoretical 7980 [m/s] speed.
Data and results are included int he Table 2.

		a-b distance	a-c distance	a-d distance
		0.015[m]	0.0212132[m]	0.0259807[m]
1st case	Time needed for total burn[10^{-6} s]	2,600280	3,500219	4,200000
	Burning speed [m/s´]	5768,610359	6060,535819	6185,895594
	Compared to the reference [%]	**72,2884**	**75,9466**	**77,5175**
2nd case	Time needed for total burn [10^{-6} s]	2,000885	2,897841	3,547241

	Burning speed [m/s´]	7496,682397	7320,346986	7324,217622
	Compared to the reference [%]	**93,9434**	**91,7337**	**91,7822**
3rd case	Time needed for total burn [10^{-6} s]	1,921741	2,744498	3,311228
	Burning speed [m/s´]	7805,421096	7729,355126	7846,261034
	Compared to the reference [%]	**97,8123**	**96,8591**	**98,3241**
4th case	Time needed for total burn [10^{-6} s]	1,901888	2,706447	3,304630
	Burning speed [m/s´]	7886,898639	7838,026208	7861,927456
	Compared to the reference [%]	**98,8333**	**98,2209**	**98,5204**

Table 2. Comparison of burning speed with theoretical value

Analysis of results lead to the following conclusions:

1. The software chosen is suitable for the analysis and presentation of the process on PC.
2. Theroretical value can be accessed by the refinement of the finite element network . Comparison of the first and second case shows that trisection of the scale leads to a 20% improvement of the precision.
3. The needed precision fineness, which permits scientific tests can be defined and also can be applied on PC.

REFERENCES

[1] C. A. Felippa: Introduction of Advanced applications of FEM http://www.colorado.edu/engineering/cas/courses.d/IFEM.d/IFEM.Ch01.d/IFEM.Ch01.Slides.d/IFEM.Ch01.Slides.pdf

[2] Lee E.L., Hornig HC, Kury 3W. Adiabatic expansion of high expiosive detonation products, UCRL-50422.: Lawrence Radiation Laboratory, University of California; 1968.

[3] Lee, E.L. and Tarver, C.M., "Phenomenological Model of Shock Initiation in Heterogeneous Explosives," Physics of Fluids 23 (12), December 1980, pp. 2362-2372

[4] Cooper, Paul W., Explosives Engineering, New York: Wiley-VCH,1196. ISBN 0-471-18636-8

[5] Benson DJ. Computational methods in Lagrangian and Eulerian hydrocodes. Comput Methods Appi Mech. Eng 1992;99:235—394.

[6] Salas, M. D. "The Curious Events Leading to the Theory of Shock Waves, Invited lecture, *17th Shock Interaction Symposium*, Rome, 4-8 September 2006

[7] Mader Ch.L., Numerical modeling of explosives and propellants, 2nd edition,CRC Press ISBN 0-8493-3149-8