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MODELLING OF DYNAMICAL SYSTEMS

Absztrakt/Abstract

A szerző célja, hogy komplex módszereket mutasson be a dinamikai rendszerek modellezésére. Jelen írás az elektromos, mechanikai és elektro-mechanikus rendszerekre, vagyis a mechatronikai rendszerekre fókuszál. A matematikai modellezés a legfontosabb fázis az automatikus rendszerek elemzésében és azok előzetes tervezésében. Szerző a számítógépes elemzés és tervezés területeire koncentrál, amely során számos példát mutat be a modellezési és irányítási problémák köréből.

Purpose of the author to give a complex set of methods applied for modeling of the dynamical systems. The more attention is paid for electrical, mechanical, and electro-mechanical systems, i.e. for mechatronical systems. Mathematical modeling is the most important phase in automatic systems' analysis, and preliminary design. Author of the paper devotes attention to computer aided analysis and design. Examples for this are taken from the wide branch of modeling and control problems.

Kulcsszavak/Keywords: *dinamikus rendszerek, irányítás, modellezés ~ dynamical systems, control, modeling*

I. INTRODUCTION

Modeling of the dynamical systems is in the focus of attention of scientists since many decades. Today there is a main motivation to have necessary information for automated control of the dynamical systems. Dynamical systems can be technical (e.g. electrical systems, mechanical systems etc.), biological (e.g. blood pressure control, insulin control etc.) ones, and also they can represent many other branches of economics, society, sciences etc. Dynamical modeling is necessary for computer aided preliminary design, too. There are many powerful tools to design a control system for the first possible scheme.

The designed system must be tested for design criteria, and in case of necessity must be re-designed. Purpose of the author is to present a typical set of possible dynamical systems applied in Mechatronics, and, in Robotics, too. There are many famous classical examples of

Mechanics, theory of electricity, Electrotechnics being involved in this paper to show how to get a complex set of dynamical characteristics of them. Solution of the analysis task is supported by MATLAB®, by SIMULINK® software, and by theirs toolboxes.

II. BRIEF HISTORY & LITERATURE OVERVIEW

Mathematical backgrounds for modeling of deterministic dynamical systems are given detailed in [2, 3], and stochastic systems are discussed in [1]. The theory of the control systems both for SISO and MIMO applications are outlined in [4, 5, 7, 8, 9]. In general, systems and signals are investigated by Pokorádi, L. [12]. Mathematical models of the turbulent air are discussed in [6, 10, 11, 14]. Computer aided simulation is supported by MATLAB [13].

III. MODELLING DYNAMICAL SYSTEMS

3.1. MODELLING OF THE "MASS-SPRING-DAMPER" MECHANICAL SYSTEM.

The dynamical model of the "mass-spring-damper" mechanical system can be seen in Figure 3.1. [2, 5, 7].
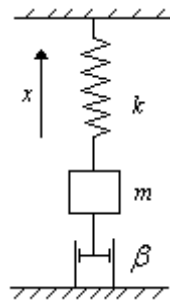

Figure 3.1. The sketch of the "mass-spring-damper" mechanical system.

Equation of motion of the free vibration system can be derived as [2, 5]:

$$\frac{d(m\mathrm{v})}{dt} = \Sigma F_i .$$
(3.1)

From Figure 3.1. it is easily can be seen that the resulting damping force is as follows [2, 3, 5]:

$$\Sigma F_i = kx + \beta \frac{dx}{dt} .$$
(3.2)

In Eq (3.2): kx is the spring force opposing to translational motion of the constant mass, m; $\beta \frac{dx}{dt}$ is the viscous damper wall friction force. Resulting equation of motion of the mechanical system as given below:

$$m\frac{d^2x}{dt^2} + \beta \frac{dx}{dt} + kx = 0 .$$
(3.3)

The solution of the second order differential equation (3.3) with constant parameters is given in [2, 3] to be as follows:

$$x(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t},$$

(3.4)

where A and B are constants derived by the initial conditions, λ_1, and λ_2 are solutions of the characteristic polynomial of

$$m\lambda^2 + \beta\lambda + k = 0.$$

(3.5)

Solving eq (3.5) yields to the following roots [2]:

$$\lambda_1 = -\frac{\beta}{2m} + \sqrt{\left(\frac{\beta}{2m}\right)^2 - \frac{k}{m}}; \quad \lambda_2 = -\frac{\beta}{2m} - \sqrt{\left(\frac{\beta}{2m}\right)^2 - \frac{k}{m}}.$$

(3.6)

Let us suppose that $\beta \neq 0$. For further discussion we suppose that

A) $\beta^2 > 4 \cdot k \cdot m$. In this particular case solutions of the eq (3.5) are real and negative ones, i.e.

$$\lambda_1 < 0, \ \lambda_2 < 0, \ |\lambda_2| > |\lambda_1|.$$

(3.7)

Let us suppose system parameters to be as follows:

Case a) $\qquad\qquad\qquad\qquad A = 1; \ B = -2; \ \lambda_1 = -1; \ \lambda_2 = -2.$

(3.8)

Case b) $\qquad\qquad\qquad\qquad A = -1; \ B = 2; \ \lambda_1 = -1; \ \lambda_2 = -2.$

(3.9)

Case c) $\qquad\qquad\qquad\qquad A = 1; \ B = 2; \ \lambda_1 = -1; \ \lambda_2 = -2.$

(3.10)

For systems defined by equations (3.8)-(3.10) computer aided simulation was done, and the results can be seen in Figure 3.2, Figure 3.3, and finally, Figure 3.4.
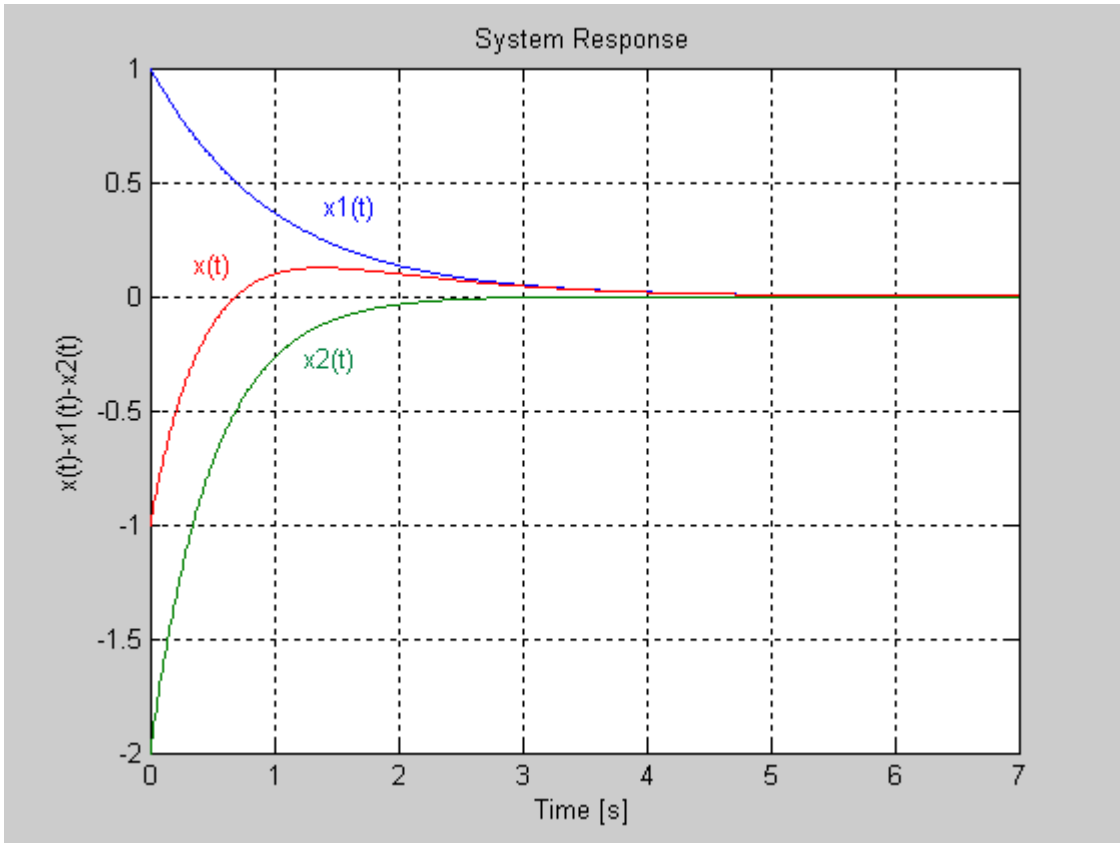
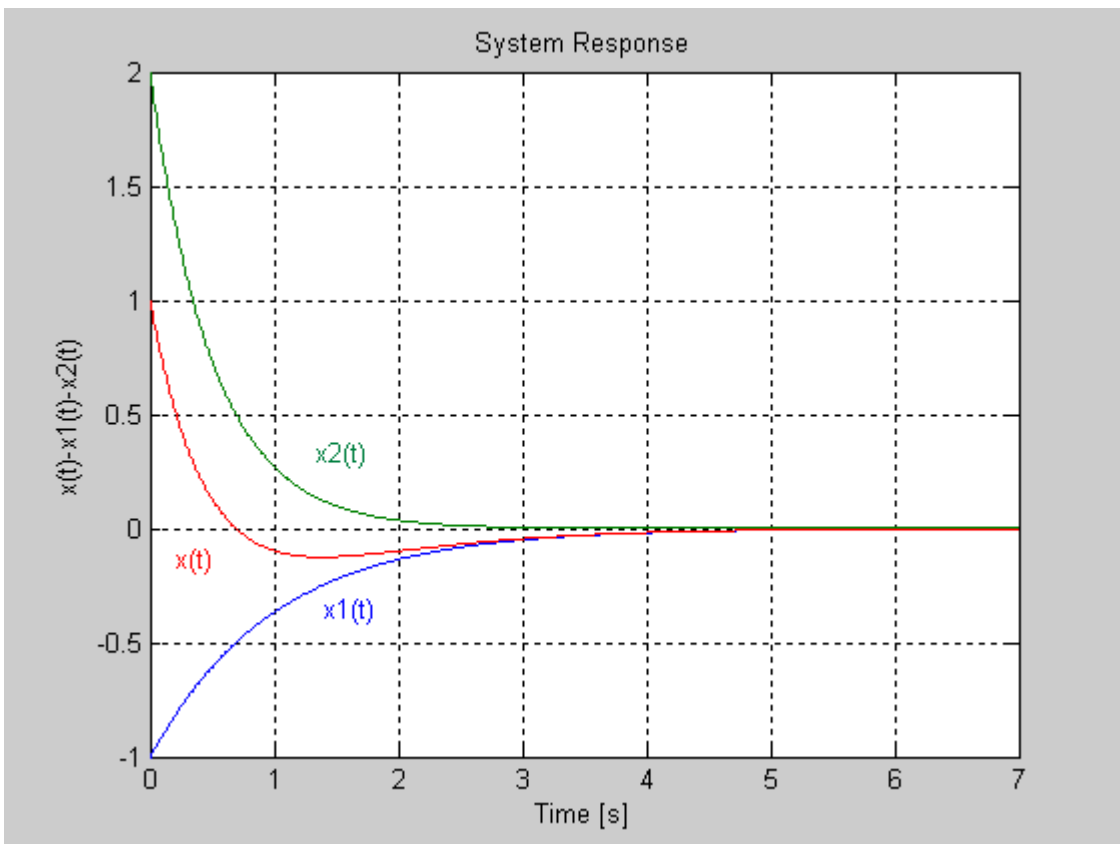Figure 3.2. Case a) $A = 1$; $B = -2$; $\lambda_1 = -1$; $\lambda_2 = -2$

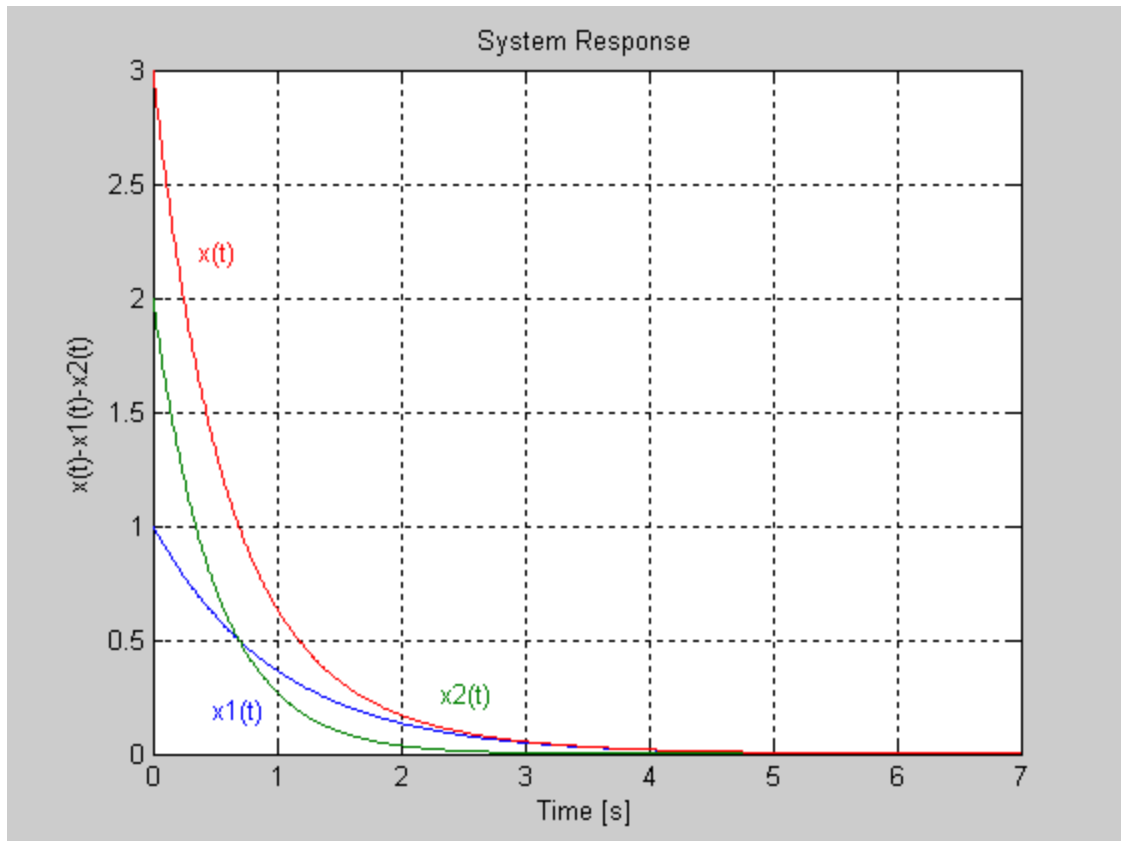Figure 3.3. Case b) $A = -1$; $B = 2$; $\lambda_1 = -1$; $\lambda_2 = -2$

Figure 3.4. Case c) $A = 1$; $B = 2$; $\lambda_1 = -1$; $\lambda_2 = -2$

From Figures 3.2.-3.4. it is easily can be derived that the mechanical system behaves aperiodically having exponential responses.

B) $\beta^2 < 4 \cdot k \cdot m$. In this particular case solutions of the characteristic polynomial (3.5) are complex conjugates ones having negative real parts. It is well-known that solution of eq (3.4) can be derived as [2, 5]:

$$x(t) = C\,e^{-\frac{\beta}{2m}t} \cos\left(\sqrt{\frac{k}{m} - \left(\frac{\beta}{2m}\right)^2}\,t\right) + D\,e^{-\frac{\beta}{2m}t} \sin\left(\sqrt{\frac{k}{m} - \left(\frac{\beta}{2m}\right)^2}\,t\right). \tag{3.11}$$

In eq (3.11) C and D are unity constant parameters depending on initial conditions. Let the mechanical system has parameter as given below:

$$\frac{\beta}{m} = 1; \quad \frac{k}{m} = 2, \quad m = 1\,kg \tag{3.12}$$

Substituting equation (3.12) into equation (3.11) yields to the next formula:

$$x(t) = e^{-0.5\,t} \cos(1.75\,t) + e^{-0.5\,t} \sin(1.75\,t) \tag{3.13}$$

Results of the computer simulation of equation (3.13) can be seen in Figure 3.5.
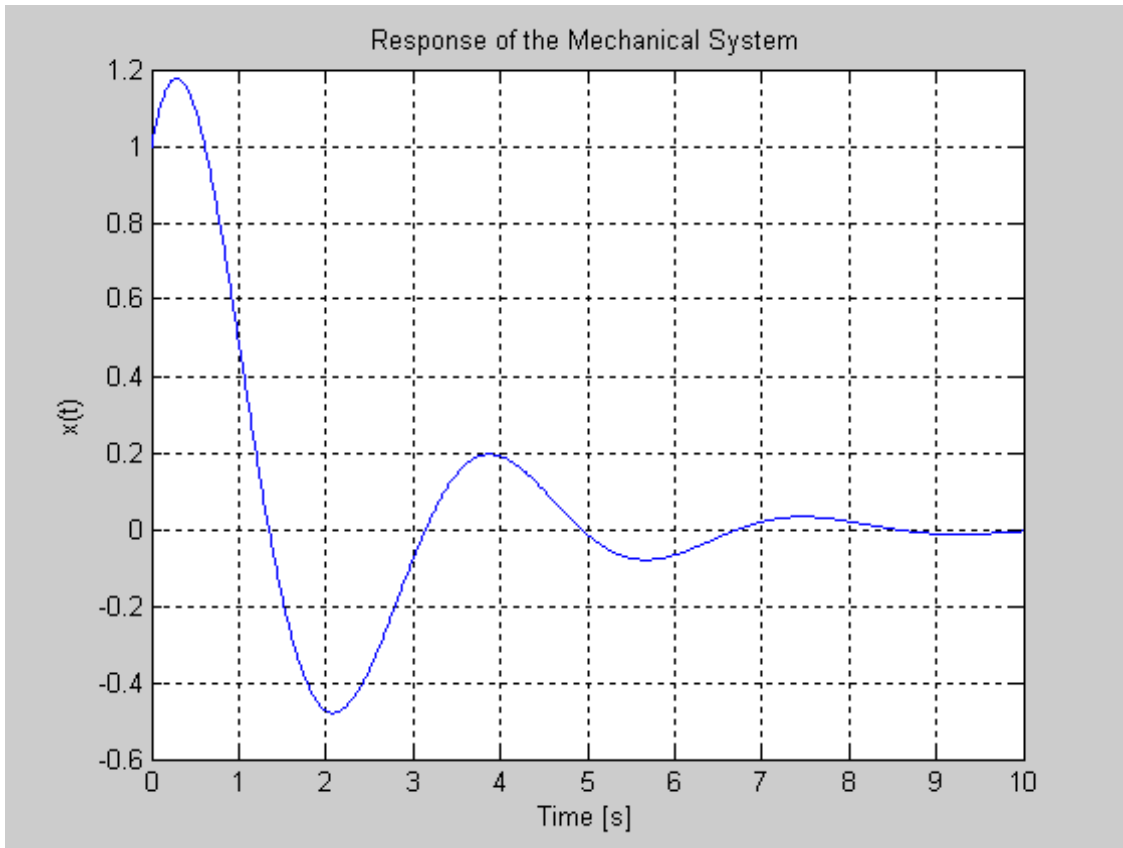
Figure 3.5. Dynamical System Response (C=1, D=1)

3.2. MODELLING OF THE "MASS-SPRING" MECHANICAL SYSTEM CONSTRAINED TO SINUSOIDAL INPUT SIGNAL

The 'mass-spring' mechanical system can be seen in Figure 3.6, where l is the spring length in the steady-state position; x is the increase of the spring length, m is the mass of body. For undamped mechanical system equation of motion can be derived as follows [2, 5]:

$$m\frac{d^2x}{dt^2} + kx = a_o \sin \omega t \qquad (3.14)$$

From Figure 3.6. coordinate of the mass measured from its basic level easily can be determined as:

$$a_o \sin \omega t + l + x \qquad (3.15)$$

Using Newton's Second Law dynamic equation (3.14) can be rewritten as:

$$m\frac{d^2}{dt^2}(a_o \sin \omega t + l + x) + kx = 0, \qquad (3.16)$$

or in other manner:

$$m\frac{d^2x}{dt^2} + kx = ma_o\omega^2 \sin \omega t = F_o \sin \omega t \qquad (3.17)$$
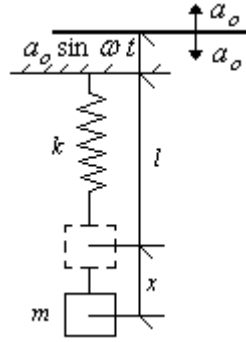
347

Figure 3.6. The "mass-spring" system.

Solution of the dynamical equation (3.14) can be derived as sum of the solution of the homogeneous equation of the linear system (3.14), and a particular solution of the inhomogeneous linear equation (3.14). Let find particular solution in the form of the next formula:

$$x = C \sin \omega t .$$ (3.18)

Substituting equation (3.18) into equation (3.17) results in

$$- mC\omega^2 + kC = F_o .$$ (3.19)

From equation (3.19) we have

$$C = \frac{F_o}{k - m\omega^2} .$$ (3.20)

Let $\omega_o = \sqrt{\frac{k}{m}}$ denote the resonance peak frequency of the mechanical system being investigated.

Thus, particular solution (3.18) can be rewritten as:

$$x = \frac{F_o}{m} \frac{\sin \omega t}{\omega_o^2 - \omega^2} = \frac{F_o}{k} \frac{\sin \omega t}{1 - \dfrac{\omega^2}{\omega_o^2}}$$ (3.21)

Solution of the dynamical equation can be found as

$$x = A\cos \omega_o t + B\sin \omega_o t + \frac{F_o}{m} \frac{\sin \omega t}{\omega_o^2 - \omega^2} ,$$ (3.22)

where constants A, and B can be found using initial conditions. Let for $t = 0$ initial conditions are as follows:

$$x(0) = 0; \quad \left. \frac{dx}{dt} \right|_{t=0} = v_o = 1.$$ (3.23)

Finally, resulting equation of (3.22) can be derived as [2, 5]:

$$x = v_o \frac{\sin \omega_o t}{\omega_o} + \frac{F_o}{m} \frac{\sin \omega t}{(\omega_o^2 - \omega^2)\omega_o} \left(\omega_o \sin \omega t - \omega \sin \omega t \right)$$ (3.24)

Dynamical system defined by equation (3.24) was constrained to computer simulation. Results can be seen in Figure 3.7.
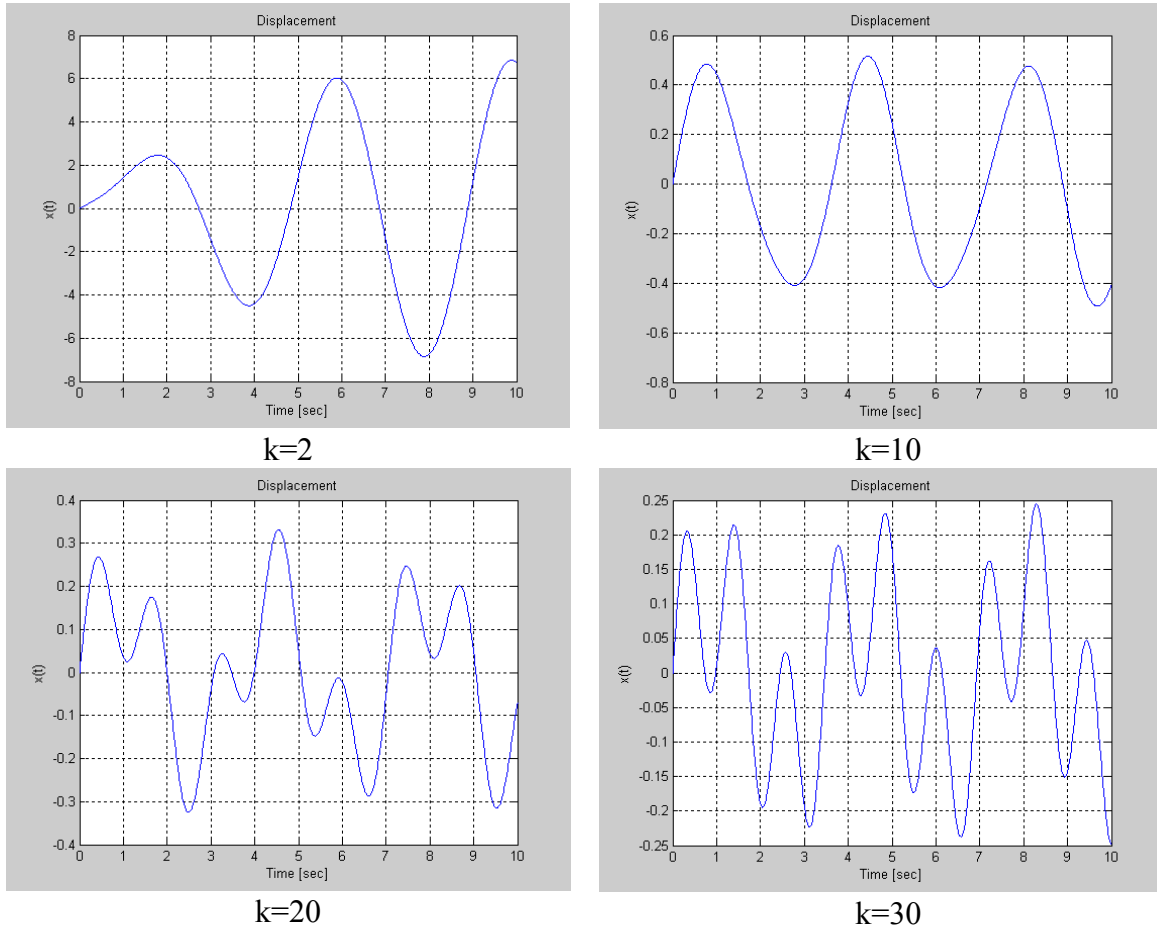
Figure 3.7. Mass-spring system displacement

For particular case of $\omega = \omega_o$ particular solution of (3.21) does not exist, and instead of we try to find this solution in the next form:

$$x = C\,t\cos\omega\,t \qquad (3.25)$$

Substituting eq (3.25) into equation (3.14) constant C can be derived as

$$-2Cm\omega_o\sin\omega_o\,t - Cmt\,\omega_o^2\cos\omega_o\,t + kCt\cos\omega_o\,t = F_o\sin\omega_o\,t \qquad (3.26)$$

Suppose that $m\,\omega_o^2 = k$, equation (3.26) can be simplified to that of

$$C = -\frac{F_o}{2m\omega_o} \qquad (3.27)$$

Thus, final form of the solution of equation (3.14) can be derived as:

$$x = A\cos\omega_o\,t + B\sin\omega_o\,t - \frac{F_o}{2m\omega_o}t\cos\omega\,t \qquad (3.28)$$

The 'spring-mass' system is often supplemented with viscous damper, i.e. equation of motion (3.14) can be re-written as follows:

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F_o\sin\omega\,t\,. \qquad (3.19)$$

349

Let find particular solution of the inhomogeneous equation of (3.19) in the following manner:

$$x = a \sin \omega t + b \cos \omega t .\qquad(3.20)$$

Constants a and b can be found with substitution of eq (3.20) into (3.19), and it yields:

$$(k - m\omega^2)a - \beta\omega\, b = F_0 ,\qquad(3.21)$$

$$(k - m\omega^2)b - \beta\omega\, a = 0 .\qquad(3.22)$$

Solving system of equations (3.21), and (3.22) for coefficients a and b, and substituting them into equation (3.20) gives the following formula:

$$x = F_0 \frac{(k - m\omega^2)}{(k - m\omega^2)^2 + \beta^2\omega^2} \sin \omega t - F_0 \frac{\beta}{(k - m\omega^2)^2 + \beta^2\omega^2} \omega \cos \omega t .\qquad(3.23)$$

It is well-known that final solution of equation (3.1) can be derived as sum of equations (3.23), and (3.4). From (3.4) it evident that (3.4) part is goes to zero while time goes to infinity, i.e. behavior of the dynamical system determined by eq (3.23).

IV. MATHEMATICAL MODELS OF THE STOCHASTIC CONTINUOUS ATMOSPHERIC DISTURBANCES

There are two powerful mathematical models of the continuous gust representations. The first is, the so-called von Kármán spectrum, which is better fit registrations of the turbulent air records. The von Kármán power spectral density (PSD) function is given below as follows [1, 6, 10, 11]:

$$\Phi_{K\acute{a}rm\acute{a}n}(\Omega) = \frac{\sigma^2 L}{\pi} \frac{1 + \frac{8}{3}(1,339 L\Omega)^2}{\left(1 + 1,339 L^2\Omega^2\right)^{11/6}} ,\qquad(4.1)$$

where L [m] is the gust wavelength, $\Omega = \omega U_0^{-1}$ [rad/m] is spatial frequency, ω [rad/s] is the observed angular frequency, and finally, σ [m/s] *r.m.s.* gust velocity.

The second one, the more favored PSD function is the Dryden PSD function, which can be programmed more easily then the von Kármán-model. If there is no structural analysis is performed the use of Dryden PSD function is permissible. The Dryden PSD function can be defined as given below [1, 6, 10, 11, 14]:

$$\Phi_{\text{Dryden}}(\Omega) = \frac{\sigma^2 L}{\pi} \frac{1 + 3L^2\Omega^2}{\left(1 + L^2\Omega^2\right)^2} .\qquad(4.2)$$

Having goal to analyze hypothetical aircraft mathematical models with no interest in investigation of the structural behavior and supposing aircraft to be rigid one, the simplest mathematical form of the PSD function defined by equation of (4.2) we will use in this article. Regarding basic references of [6, 10, 11, 14] one can define PSD functions of the component speed of the turbulent air along body axis system of the aircraft, i.e.:

$$\Phi_{u_g}(\Omega) = \frac{2\sigma_u^2 L_u}{\pi} \frac{1}{1 + (L_u\Omega)^2}\qquad(4.3)$$

$$\Phi_{v_g}(\Omega) = \frac{\sigma_v^2 L_v}{\pi} \frac{(1 + 3(L_v\Omega)^2)}{\left[1 + (L_v\Omega)^2\right]^2}\qquad(4.4)$$

$$\Phi_{w_g}(\Omega) = \frac{\sigma_w^2 L_w}{\pi} \frac{(1 + 3(L_w \Omega)^2)}{\left[1 + (L_w \Omega)^2\right]^2} \tag{4.5}$$

where $\sigma_i^2 = \int_0^\infty \Phi_i(\Omega)\, d\,\Omega_i \big|_{i=u,v,\,or\,w}$. Since $\omega = U_o \Omega$ formulas of (4.3)–(4.5) may be rewritten as follows:

$$\Phi_{u_g}(\omega) = \frac{2\sigma_u^2 L_u}{U_o \pi} \frac{1}{\left\{1 + (L_u / U_o)^2 \omega^2\right\}}, \tag{4.6}$$

$$\Phi_{v_g}(\omega) = \frac{\sigma_v^2 L_v}{U_o \pi} \frac{(1 + 3(L_v / U_o)^2 \omega^2)}{\left\{(1 + (L_v / U_o)^2 \omega^2)^2\right\}}, \tag{4.7}$$

$$\Phi_{w_g}(\omega) = \frac{\sigma_w^2 L_w}{U_o \pi} \frac{(1 + 3(L_w / U_o)^2 \omega^2)}{\left\{(1 + (L_w / U_o)^2 \omega^2)^2\right\}}. \tag{4.8}$$

For generating random signals with the required intensity, scale length, and PSD functions for given speed and height of the flight, a hypothetical wide-band noise generator with PSD function of $\Phi_N(\omega)$ must be used to provide signal with the linear filter, chosen such that it has an appropriate frequency response so that the output signal from the linear filter will have a PSD function of $\Phi_i(\omega)$ (see Figure 4.1.) [6]:

$$\Phi_i(\omega) = \left| G_i(s) \right|^2_{s=j\omega} \Phi_N(\omega) = G_i(s)\, G_i(-s) \big|_{s=j\omega} \Phi_N(\omega). \tag{4.9}$$

If the white noise source is chosen so that its power spectrum is similar to that of called 'white' noise one can write that

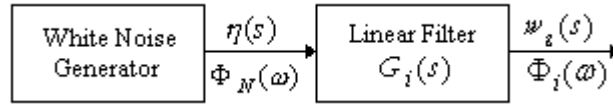$$\Phi_N(\omega) = 1. \tag{4.10}$$

Figure 4.1. Block Diagram for Generating Stochastic Signals.

Substituting equation (4.10) into equation of (4.9) result the following formula

$$\Phi_i(\omega) = \left| G_i(s) \right|^2_{s=j\omega} \Phi_N(\omega) = G_i(s)\, G_i(-s) \big|_{s=j\omega}. \tag{4.11}$$

The linear filter transfer functions of $G_i(s)$ are given in [6] to be:

$$G_{u_g}(s) = \frac{\sqrt{K_u}}{s + \lambda_u}, \qquad G_{v_g}(s) = \sqrt{K_v} \frac{s + \beta_v}{(s + \lambda_v)^2}, \qquad G_{w_g}(s) = \sqrt{K_w} \frac{s + \beta_w}{(s + \lambda_w)^2}, \tag{4.12}$$

where:

$$K_u = \frac{2U_o \sigma_u^2}{L_u \pi}, \quad K_v = \frac{3U_o \sigma_v^2}{L_v \pi}, \quad K_w = \frac{3U_o \sigma_w^2}{L_w \pi}, \tag{4.13}$$

$$\beta_v = \frac{U_o}{\sqrt{3} L_v}, \quad \beta_w = \frac{U_o}{\sqrt{3} L_w}, \tag{4.14}$$

$$\lambda_u = \frac{U_o}{L_u}, \quad \lambda_v = \frac{U_o}{L_v}, \quad \lambda_w = \frac{U_o}{L_w}. \tag{4.15}$$

It is easily can be derived that substitution equations (4.12)–(4.15) into equation (4.9) results in the PSD functions of the Dryden-models's PSD-functions of (4.6)–(4.8). If the air

turbulence model is used for analysis of its effects on flight of the small UAV aircraft let the initial parameters be as they are given below:

$$H = 100\,m \cong 328{,}084\;feet;\quad U_0 = 25\;m/s = 90\;km/h^2. \qquad (4.16)$$

From equations (4.13)–(4.15) it is evident that for derivation of transfer functions of the linear filters defined by equation (4.12) it is necessary to know turbulence scale of L_i, and turbulence intensity of σ_i, measured along appropriate axis of the given coordinate system. Let us consider NASA-parameters taken from [6, 10] to be as follows:

- along longitudinal (OX) axis: $3{,}4\;m/s \le \sigma_u \le 0{,}85\;m/s$ (4.17)
- along lateral (OY) axis: $2{,}8\;m/s \le \sigma_v \le 0{,}7\;m/s$,   (4.18)
- along vertical (OZ) axis: $1{,}8\;m/s \le \sigma_w \le 0{,}45\;m/s$.   (4.19)

For extreme weather conditions (thunderstorm) MCLEAN [6] suggests turbulence intensities as they given below:

$$\sigma_u = \sigma_v = \sigma_w = 7\;m/s. \qquad (4.20)$$

Turbulence integral scale lengths L_i of the low altitude turbulence models for $10\;feet \le h \le 1000\;feet$ can be derived using following formulas [6, 10]:

$$L_u = 2L_v = \frac{h}{(0{,}177 + 0{,}000823 \cdot h)^{1{,}2}},\quad L_w = 0{,}5\,h. \qquad (4.21)$$

Regarding MCLEAN, for extreme weather conditions (thunderstorm) one can apply following integral scale lengths given in [6]:

$$L_u = L_v = L_w = 580\;m. \qquad (4.22)$$

Constant speed components of the turbulent air are given in military standards of [4, 7] as function of theirs exceedance. For the low altitude random turbulence models intensity of the turbulence, σ_w can be measured as [6, 11, 14]:

$$\sigma_w = 0{,}1\,u_{20}, \qquad (4.23)$$

where u_{20} is constant longitudinal component speed of the turbulent air measured at the altitude of $h = 20\;feet$. Using equations of (4.21)–(4.22) integral scale lengths of the air turbulence were found and they are summarized in Table 1.

Table 1. Integral scale lengths at altitude of $H = 100\,m \cong 328{,}084\;feet$.

Scale length, [m]	Nominal (Nom)	Extreme (Thunderstorm)
L_u	862,185497 feet $\cong$ 262,7941311 m	580
$L_v = 0{,}5\,L_u$	431,0927485 feet $\cong$ 131,3970655 m	580
L_w	50	580

Using equations of (4.17)–(4.20) turbulence intensities were found and they are summarized in Table 2.

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[2] $1\;foot \cong 0{,}3048\;m$ — $1\;m \cong 3{,}28084\;feet$

Table 2. Turbulence intensities.

Turbulence intensities	NASA-Min (Min)	NASA-Max (Max)	Extreme (Thunderstorm)
σ_u, [m/s]	0,85	3,4	7
σ_v, [m/s]	0,7	2,7	7
σ_w, [m/s]	0,45	1,8	7

Constant longitudinal component speed of the turbulent air, called u_{20}, were found using military standard of [10], and using equations of (4.21)–(4.22). Constant speed of u_{20} are summarized in Table 3.

Table 3. Constant speed of u_{20}.

Turbulent Air Characteristics	NASA-Min (Min)	NASA-Max (Max)	Extreme (Thunderstorm)
$\sigma_w = 0{,}1\,u_{20}$, [m/s]	0,45	1,8	7
u_{20}, [m/s] – [km/h]	4,5 – 16,2	18 – 64,8	70 – 252

Linear transfer functions defined by equations (4.12) having parameters given by equations of (4.13)–(4.15), and satisfying conditions derived by equations (4.16)–(4.24), and considering weather conditions given by Table 1., and Table 2, can be determined, and they can be found in the following tables given below [11, 14]:

Table 3. Parameters of the linear filters providing longitudinal speed component of the air turbulence, $u_g(t)$.

Filter Parameters		
Weather Conditions	$K_u = \dfrac{2\,\sigma_u^2 U_o}{L_u \pi}\ \left[\dfrac{m^2}{s^3} \right]$	$\lambda_u = \dfrac{U_o}{L_u}\ \left[s^{-1} \right]$
NASA-Min	0,043756496	0,095131547
NASA-Max	0,700103937	0,095131547
Extreme (Thunderstorm)	1,344584864	0,043103448

Table 4. Parameters of the linear filters providing lateral
speed component of the air turbulence, $v_g(t)$.

Filter Parameters			
Weather Conditions	$K_v = \dfrac{3\sigma_v^2 U_o}{L_v \pi}\ \left[m^2 \middle/ s^3 \right]$	$\beta_v = \dfrac{U_o}{\sqrt{3}L_v}\ \left[s^{-1} \right]$	$\lambda_v = \dfrac{U_o}{L_v}\ \left[s^{-1} \right]$
NASA-Min	0,089027057	0,109848449	0,190263095
NASA-Max	1,324504595	0,109848449	0,190263095
Extreme (Thunderstorm)	8,902705783	0,024885787	0,043103448

Table 5. Parameters of the linear filters providing vertical
speed component of the air turbulence, $w_g(t)$.

Filter Parameters			
Weather Conditions	$K_w = \dfrac{3\sigma_w^2 U_o}{L_w \pi}\ \left[m^2 \middle/ s^3 \right]$	$\beta_w = \dfrac{U_o}{\sqrt{3}L_w}\ \left[s^{-1} \right]$	$\lambda_w = \dfrac{U_o}{L_w}\ \left[s^{-1} \right]$
NASA-Min	0,096686627	0,288675134	0,5
NASA-Max	1,546986047	0,288675134	0,5
Extreme (Thunderstorm)	2,016877296	0,024885787	0,043103448

Using parameters of Table 3, Table 4, Table 5, transfer functions of the linear filters
defined by equation (4.12) can be derived as follows [11, 14]:

$$G_{u_g}^{Min}(s) = \frac{0,20918}{s + 0,09513}, \tag{4.24-1}$$

$$G_{u_g}^{Max}(s) = \frac{0,83672}{s + 0,09513}, \tag{4.24-2}$$

$$G_{u_g}^{Extr}(s) = \frac{1,15956}{s + 0,04310}, \tag{4.24-3}$$

$$G_{v_g}^{Min}(s) = 0,29837 \ \frac{s + 0,10984}{s^2 + 0,38052\,s + 0,03620} \tag{4.25-1}$$

$$G_{v_g}^{Max}(s) = 1,15087 \ \frac{s + 0,10984}{s^2 + 0,38052\,s + 0,03620}, \tag{4.25-2}$$

$$G_{v_g}^{Extr}(s) = 2,98374 \ \frac{s + 0,02488}{s^2 + 0,08620\,s + 0,00186} \tag{4.25-3}$$

$$G_{w_g}^{Min}(s) = 0,31094 \ \frac{s + 0,28867}{s^2 + s + 0,25}, \tag{4.26-1}$$

$$G_{w_g}^{Max}(s) = 1,24377 \ \frac{s + 0,28867}{s^2 + s + 0,25} \tag{4.26-2}$$

$$G_{w_g}^{Extr}(s) = 1{,}42016 \; \frac{s + 0{,}02488}{s^2 + 0{,}08620\,s + 0{,}00185}$$ (4.26-3)

Using linear transfer function models of equations (4.24)–(4.26) it is easy to generate random time series with given statistical parameters, which can be applied both for identification, modeling, analysis and design purposes [6, 10, 11, 14].

4.1. RESULTS OF THE COMPUTER SIMULATION

Using principle derived by Figure 1., and using transfer functions of the linear filters defined for several weather conditions one can generate computer code for solution of this problem. In our preliminary study we have used MATLAB® R2009 [13]. Regarding mathematical models of the random air outlined in Chapter 4 all components of the speeds of the turbulent air measured along axes of the aircraft body-axis system, and they will be presented in the next sections.

4.1.1. RANDOM LONGITUDINAL SPEED COMPONENT OF THE TURBULENT AIR

The longitudinal speed component is very important from the point of view of the basic flight conditions, i.e. aircraft flight is limited with its minimum longitudinal speed of, say, $u_{\min}$. From Chapter 3 it is known that equilibrium speed of the hypothetical UAV aircraft is $u_o = 25\,m/s$. Result of the computer simulation can be seen in Figure 4.2.

From Figure 4.2, it is easily can be determined that in time domain of $(50 \div 100)$ seconds, in other words, in the root of the turbulent zone, the mean value of the longitudinal speed is approximately, $u_{mean} \cong 4{,}2\,m/s$, which is 16,8 % of that of the equilibrium one. There is a question arising from analysis of the characteristics of the longitudinal speed component direction, i.e. it can be coinciding one to that of the mean direction of the flight, or it can oppose aircraft flight

In other words, longitudinal speed component of the turbulent air can be called for head-wind, or, tail wind. Going that way, longitudinal speed of the aircraft flying through atmospheric turbulence can be derived as follows:

for "head-wind": $u_{head} = u_o - u_{mean} = 25 - 4{,}2 = 20{,}8\,m/s$, (4.27)

for "tail wind": $u_{tail} = u_o + u_{mean} = 25 + 4{,}2 = 29{,}2\,m/s$. (4.28)
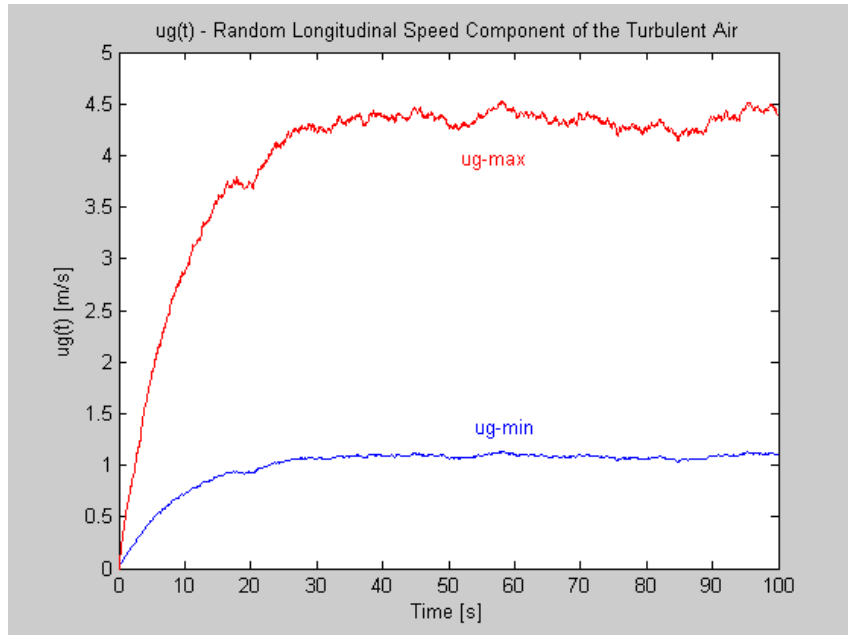
Figure 4.2. Longitudinal Speed Component of the Stochastic Air.

4.1.2. RANDOM LATERAL SPEED COMPONENT OF THE TURBULENT AIR.

Using the same manner as it was shown in previous section, computer code for random lateral speed component of the turbulent air was generated, and results of the computer simulation can be seen in Figure 4.3. From Figure 4.3. it is easily can be seen that in the time domain of about (50÷100) seconds, the mean values of the lateral speed are:

$$v_{max} \cong 1,7 \, m/s, \ v_{min} \cong 0,5 \, m/s . \qquad (4.3)$$

If to suppose weather conditions having statistical parameters between weather conditions of NASA-Min, and NASA-Max, it can be supposed that mean value of the lateral speed is, approximately, of 1 m/s.
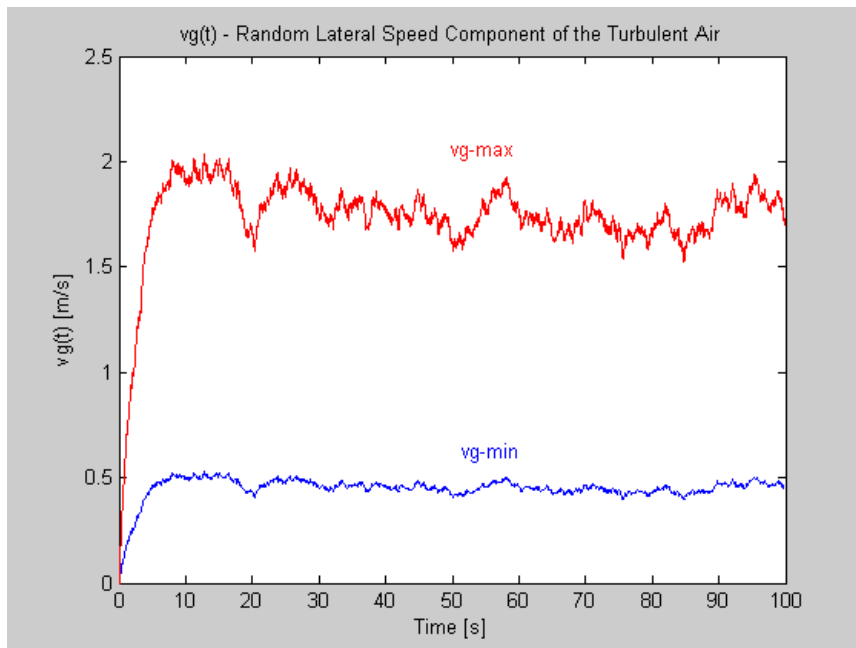

Figure 4.3. Lateral Speed Component of the Stochastic Air.

356

It means that during flight aircraft changes it lateral coordinate for about $\cong 4$ m in one second.

If to take into consideration the free-flight of the aircraft, or even if in normal flight aircraft "pilot" does not corrects the lateral coordinate, in 50 seconds time period, being investigated above, aircraft maintains distance of 1250 m, changing its lateral coordinate for 200 m. It is obvious, that there is a strong need to compensate lateral deviation measured from the flight direction.

4.1.3. RANDOM VERTICAL SPEED COMPONENT OF THE TURBULENT AIR.

Random vertical speed of the turbulent air is very important from many aspects of the altitude control of the aircraft, from the point of view of the modeling of the aeroelastic structural motion of the fuselage, and wings. There are many other reasons highlighting importance of the knowledge of the stochastic vertical speed of the atmospheric turbulences. Results of the computer simulation including NASA-Min, and NASA-Max weather conditions can be seen in Figure 4.4. From Figure 4.4. it is easily can be seen that in the time domain of about $(50 \div 100)$ seconds, the mean values of the vertical speed are as follows:

$$w_{\max} \cong 0,7 \, m/s \, , \; w_{\min} \cong 0,2 \, m/s \, . \tag{4.4}$$

It to take mean value of the vertical random speed of the wind to be of 0,5 m/s, during flight aircraft changes it altitude for 1,8 m per second. For the free-flight of the aircraft, or even if in normal flight aircraft "pilot" does not corrects the height of the flight, in 50 seconds time period, being investigated above, aircraft maintains distance of 1250 m, changing its height of the flight for 90 m, to that of the initial of $H_o \cong 100 \, m$. It means that having no control on aircraft altitude, in turbulent air aircraft nearly duplicates its height of the flight. It is obvious, that height of the flight must be controlled, and altitude must be kept at its constant value.
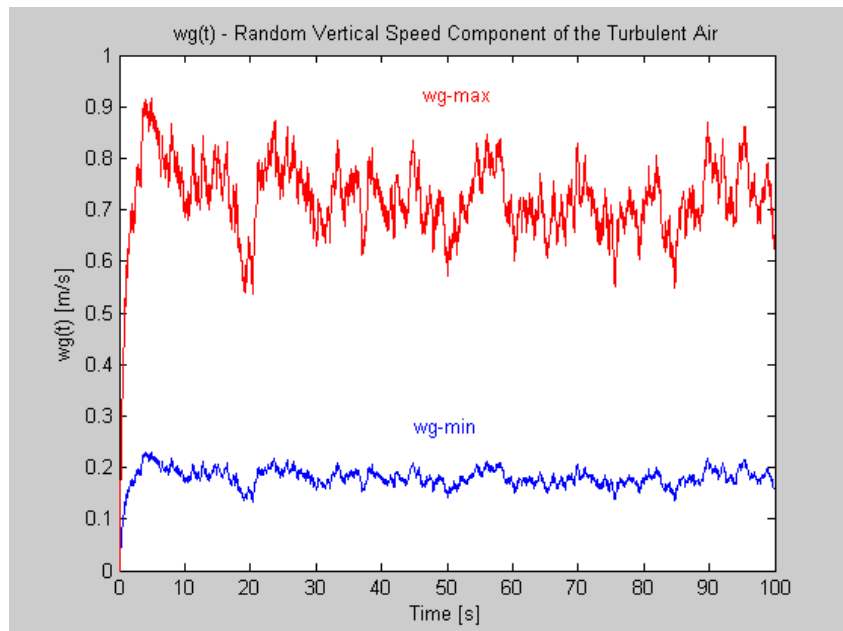

Figure 4.4. Vertical Speed Component of the Stochastic Air.

4.1.4. RESULTS OF THE COMPUTER SIMULATION OF THE ATMOSPHERIC TURBULENCES FOR THE "NASA-MIN" WEATHER CONDITIONS

Using results of the previous computer simulation, for "NASA-Min" weather conditions all appropriate time series of the longitudinal, lateral, and vertical components of the random air were plot in one, common coordinate system, and they can be seen in Figure 4.5.
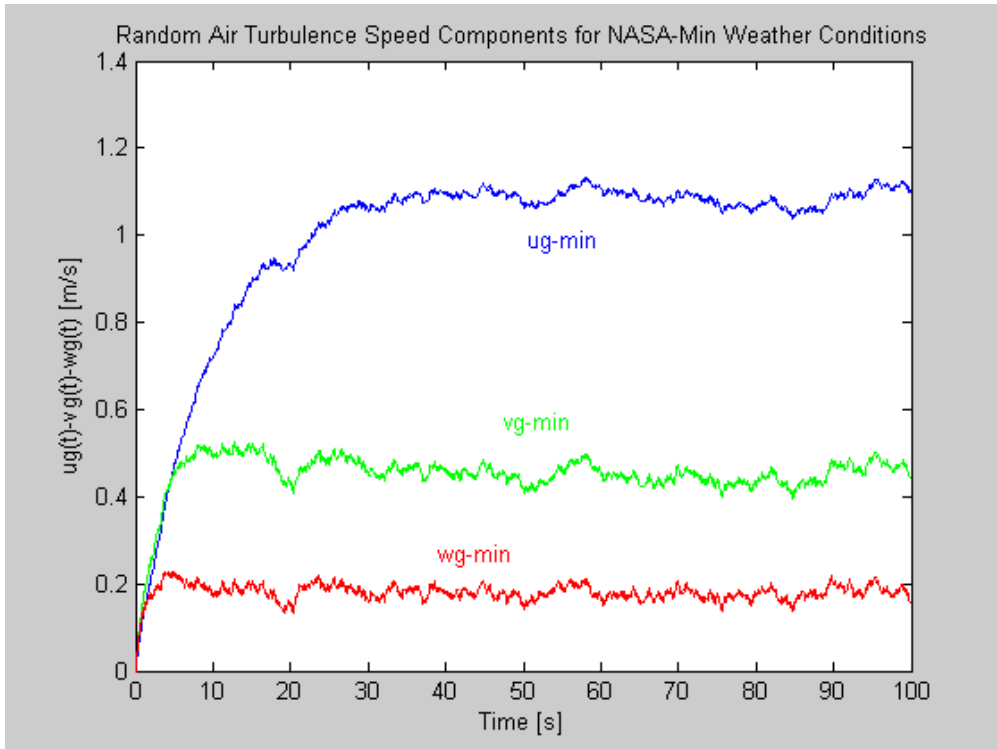

Figure 4.5. Results of the Computer Simulation for "NASA-Min" Weather Conditions.

4.1.5. RESULTS OF THE COMPUTER SIMULATION OF THE ATMOSPHERIC TURBULENCES FOR THE "NASA-MAX" WEATHER CONDITIONS

Using results of the computer simulation made before, for "NASA-Max" weather conditions all appropriate time series of the longitudinal, lateral, and vertical components of the random air were plot in one, common coordinate system, and they can be seen in Figure 4.6.
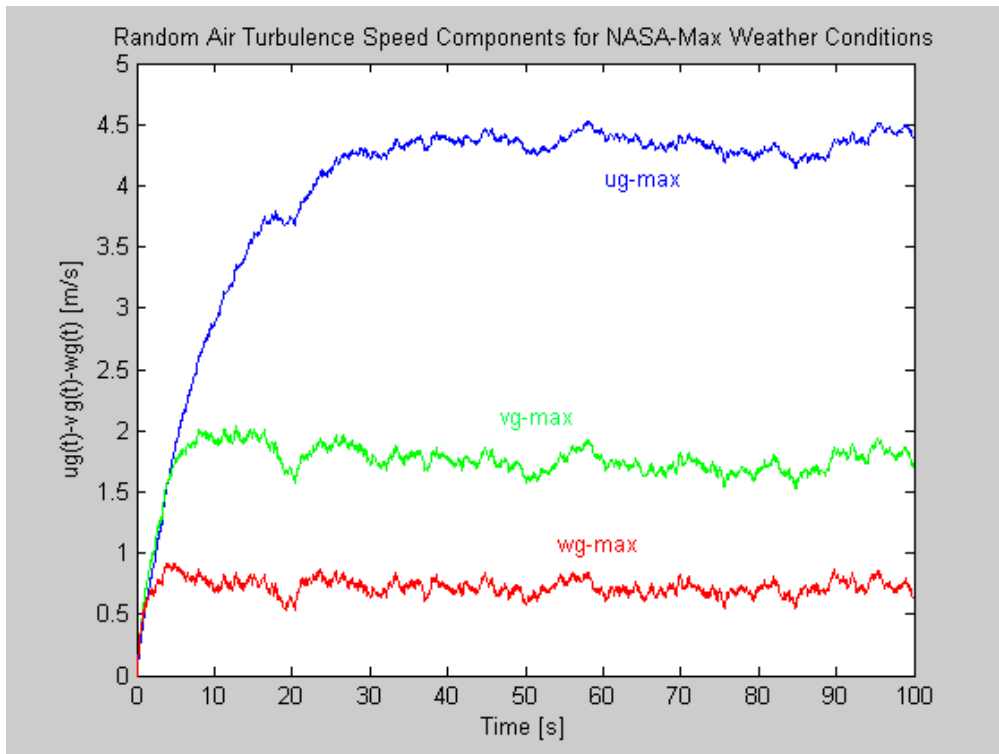
Figure 4.6. Results of the Computer Simulation for "NASA-Max" Weather Conditions.

From Figure 4.6. it is easily can be derived that longitudinal speed component, $u_g(t)$, of the atmospheric turbulence has largest mean value. It is evident that for head-wind weather conditions, there is exists a maximum value of the longitudinal random speed, $u_{g_{max}}(t)$, which is allowed to avoid stalling of the aircraft.

4.1.6. RESULTS OF THE COMPUTER SIMULATION OF THE ATMOSPHERIC TURBULENCES FOR THE "EXTREME – THUNDERSTORM" WEATHER CONDITIONS

Result of these computer simulations are mainly hypothetical, however, it is necessary to know how extreme air masses are moving. These results are very important although from the point of view of the flight achieved beyond visual range for large distances, when there are big differences between weather conditions at arrival and departure airfields. Result of the computer simulation can be seen in Figure 4.6.

The most important result is that atmospheric turbulence has largest value in the mean of lateral component of the turbulent air. The other important statement coming form this analysis, that if to consider maximum value of the longitudinal head-wind to be of $u_{g\_head}(t) = 5\,m/s$, this maximum value is reached at about 5 seconds of the computer-aided simulation. It means that to avoid stalling of the aircraft it is necessary to compensate decrease of the longitudinal speed of the aircraft increasing throttle, or it is necessary to maintain maneuver to keep given flight parameters in the defined flight envelope of the given type of the aircraft.
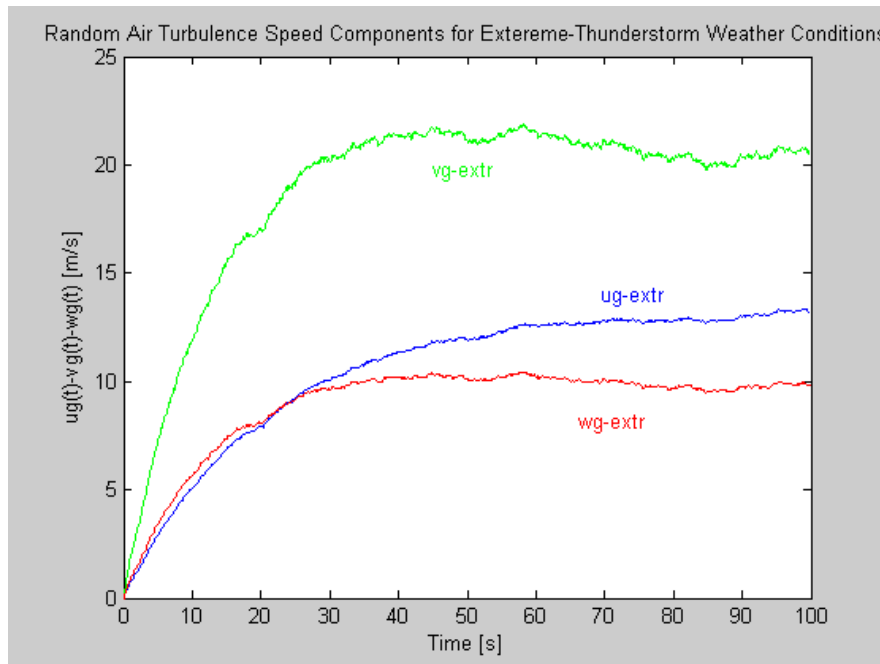
Figure 4.6. Results of the Computer Simulation for "Extreme" Weather Conditions.

The most important result is that atmospheric turbulence has largest value in the mean of lateral component of the turbulent air. The other important statement coming form this analysis, that if to consider maximum value of the longitudinal head-wind to be of $u_{g\_head}(t) = 5\,m/s$, this maximum value is reached at about 5 seconds of the computer-aided simulation. It means that to avoid stalling of the aircraft it is necessary to compensate decrease of the longitudinal speed of the aircraft increasing throttle, or it is necessary to maintain maneuver to keep given flight parameters in the defined flight envelope of the given type of the aircraft.

V. SUMMARY

Mathematical models are widely used during preliminary analysis, identification, and design of the automatic control systems. They can be both deterministic and random ones, regarding theirs time domain behavior. They support system description whether it is continuous, or discrete. Modeling of dynamical systems is supported by many computer packages e.g. MATLAB®, SIMULINK®, and theirs toolboxes.

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